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Quiz 2 Doc file

Due 4/23/2014

#1) i) (∀x)(S(x) --> B(x))

ii) ∃x (B(x) ^ ¬S(x))

iii) ¬∀x (S(x) ^ B(x) ^ ¬A(x))

#2) 1) All good athletes are either good soccer players or good basketball players

2) Not Everyone is a good basketball player

3) There are people that are either a good soccer player and a bad basketball player or is not a good athlete

4) There isn't anybody that is a good soccer player and a bad basketball player

#3) Show that ¬∀x(P(x) → Q(x)) and ∃x(P(x)∧¬Q(x)) are logically equivalent.

¬∀x(P(x) → Q(x)) and ∃x(¬(P(x)∧¬Q(x)))

are logical equivalence from De Morgan's law for universal quantifiers. We also have that

¬(P(x) --> (Q(x)) and P(x) ^¬Q(x) are logical equivalence for all x's by using the logical equivalence table. Then it follows that ¬∀x(P(x) → Q(x)) and ∃x(P(x)∧¬Q(x)) are logically equivalent since we are able to join the two logically equivalent identities together.

#8) Use a direct proof to show that n2 + n + 1 is odd for all n , where n is a natural number.  Hint: use odd and even cases.

(P → Q) : Let P be "n is an odd integer" , and Q be "n2 + n + 1 is odd for all n"

By the definition of an odd integer, it follows that n = 2k +1, where k is some integer. squaring both sides of the equation, we get n2 = (2k + 1 )(2k + 1) = 2(2k2 + 2k) + 1.

Substituting this in to our equation original equation, we get 2(2k2 + 2k) + 1 + (2k + 1) + 1.

= 2(2k2 + 2k) + 2k + 3.

By the definition of odd integer, we can conclude that n2 + n + 1 is odd (adding an odd number to an even number will always result in an odd number, in our case adding 3 to the even number).

#9) Use a proof by contraposition to show that if n3 + 5 is even, then n is odd for all natural numbers. ( Direction : You have to use proof by contraposition)

P(n) → Q(n) : Let P be "n3 + 5 is even", and Q be "n is odd for all natural numbers".

To prove by contraposition, we must show that ¬Q --> ¬P. Let's assume that n is even. By the definition of even integer, n = 2k. We substitute 2k for n, we get 2k3 + 5. By the definition of odd integers, adding an odd integer (5 in our case) to an even integer (2k3) in our case will result in an odd number, which satisfies ¬P. Thus, we have proved that the contraposition ¬Q --> ¬P is true, which means it is logically equivalent to our original condition, making it true as well.